# Dancing to a new algorithm

## Wallflowers no more, mathematicians are discovering their talents are in big demand in a wide variety of applications far beyond their discipline

*by Léo Charbonneau*

Waiting in line is a fact of life – at the doctor's office, at the checkout counter, in traffic, and even on the telephone. All of these kinds of line-ups have mathematical properties, the study of which is called queueing theory. "When you think about it, it sounds like there isn't much to study," says Myron Hlynka, a queueing theory devotee and mathematician at the University of Windsor. "But in fact, there is a lot of mathematics there. A lot of the elements are random, but randomness can be modelled mathematically."

Dr. Hlynka's work is just one example of applied mathematics, a field of study that's enjoying a real renaissance at Canadian universities. Never known as a topic to ignite cocktail conversations, it is nevertheless gaining renewed interest and appreciation among a wide variety of disciplines and particularly within industry. The biomedical sciences, trading and finance, information technology and manufacturing are just some of the areas where mathematicians are making major contributions to solving tangible problems.

With the aid of a graduate student, Dr. Hlynka has used the mathematics of queueing theory to predict the effects of drugs and exercise on cholesterol, and he is currently working on a queueing model of the Ontario court system. Dr. Hlynka also runs a queueing theory Web site (www2.uwindsor.ca/~hlynka/queue.html), which has received numerous requests for help, including one from the Metropolitan Opera in New York on ways to devise an efficient ticket exchange procedure, and a request from a golf club on how to maximize the number of members without creating unacceptable waiting lists for a tee-off time.

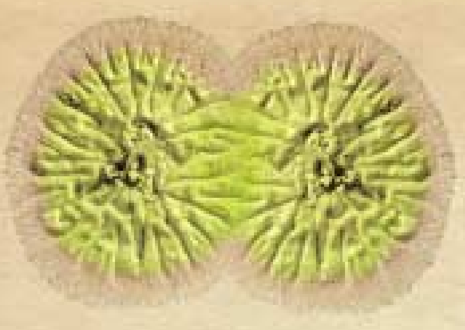
"I think increasingly it's recognized in more and more fields that mathematics is absolutely essential and fundamental," says Alan George, the former dean of mathematics at the University of Waterloo and currently its interim vice-president, academic, and provost. The boom in applied mathematics, he says, is in part due to the tremendous rise in computing power, which itself could not have been accomplished without mathematics.

As a result, in an increasing number of areas, researchers have the ability to do very large-scale computations, "and at the root of all modelling, whether it's biology or option pricing in the market, or any of the more conventional areas that you might think of – structural analysis, building airplanes or bridges, or tracking pollution transport in the ground – all of those things require sophisticated mathematics," Dr. George says.

The appeal of applied mathematics may also be partly fuelled by a desire among many graduates to seek practical applications for their research. "One way to attract funding and get attention to supporting mathematics is to be proactive and to be seen to be doing things that are important to furthering the national economy," he says.

When researchers speak of applied mathematics, it is in contrast to pure mathematics, but the line between the

Jason H. Statts



$$\underbrace{\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

*fig. 1*

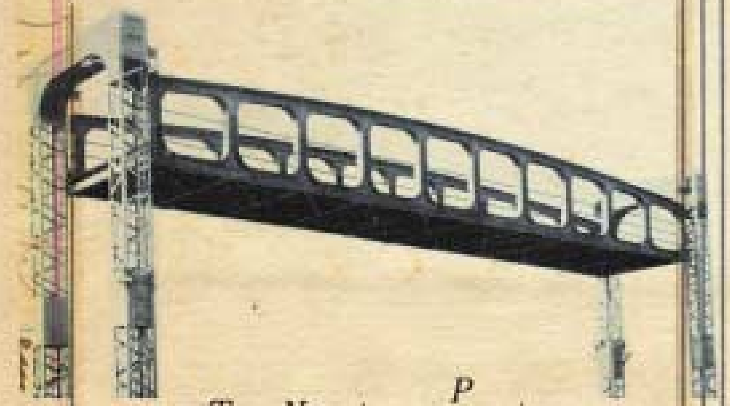


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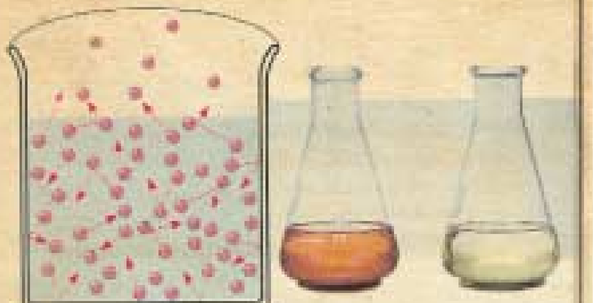
*fig. 2*



$$\underbrace{\frac{760\ \text{mm-Hg} \times 1\ \text{L}}{300\ \text{K}} = \frac{300\ \text{mm-Hg} \times V_2}{240\ \text{K}}}$$

*fig. 3*

$$\underbrace{E_k = \frac{1}{2}I_0\omega^2 + \frac{1}{2}mV^2}$$

*fig. 4*

two is blurred, says David Earn, professor of applied mathematics at McMaster University. “Even somebody working on what appears to be esoteric pure math may prove something that turns out to be of applied interest, or relevant to some specific applied problem. And it also goes the other way. Work in applied mathematics, and science in general, can sometimes stimulate great progress in pure mathematics.

“In my own mind,” says Dr. Earn, “pure mathematics is mathematics done for its own sake, whereas applied mathematics is done primarily to make progress in another field. In my case, I am using mathematics to make progress in biological problems.”

## Protecting species

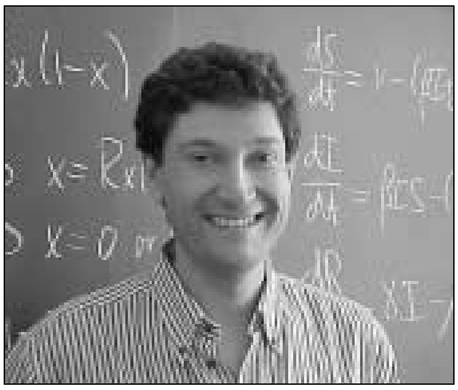
Dr. Earn recently applied mathematics to the problem of endangered species, specifically on the use of conservation corridors to help protect species that are at risk due to habitat fragmentation. In a paper published in the Nov. 17, 2000 edition of *Science* magazine, he and his colleagues presented mathematical results which can help to determine whether or not such corridors are of benefit to the targeted species.

Conservation corridors are meant to make it easier for members of a given species to move between habitat patches that may have become disconnected due to urban expansion, deforestation or other developments. The theory is that if the species is facing extinction in one area, the corridor will allow that area to be repopulated from another area, called the “rescue effect,” Dr. Earn explains. Unfortunately, corridors can have the opposite effect, putting an entire species at risk rather than confining the problem to what would otherwise have been an isolated area.

“The key point is synchrony. Corridors have the potential to synchronize population fluctuations, and consequently to cause local extinctions to happen at the same time everywhere. That would be great if we were dealing with pest species, but disastrous for species we want to preserve.” The mathematical results “give us a tool to estimate the effects of altering these dispersal patterns,” he says. “We suggest that it would be worthwhile doing such calculations before actually going about constructing corridors.”

Dr. Earn’s main area of research is the development and analysis of mathematical models of biological systems, particularly patterns of epidemics. This work is “strongly related mathematically” to the corridors analysis, in that the goal in this instance is indeed to eradicate the infectious species. Much of Dr. Earn’s recent research has been done together with biologists. “We have very different skills, so we can make different kinds of contributions to the same research problems. Both biologists and mathematicians benefit enormously from this collaboration.”

Leon Glass, a professor of physiology at McGill University, is also applying mathematics to the study of biology. In collaboration with McGill colleagues Alvin Shrier and Michael Guevara, he designs models drawn from the field of mathematics known as nonlinear dynamics to understand the patterns of irregular heart beats that occur during atrial fibrillation. Their work aims to understand the underlying mechanisms of these dangerous cardiac arrhythmias to better control them with pacemakers, drugs or surgery.



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Dr. Glass says there is a push from the U.S. granting agencies to encourage the use of mathematics in the biological sciences, and he notes that at the recent American Mathematics Society in New Orleans there was a special session devoted to mathematical modeling and physiology.

This isn’t to say Canadians aren’t making their mark. The Society for Mathematical Biology, whose current president is Alan Hastings at the University of California, selected its two previous presidents from Canada: Dr. Glass, and before that Leah Keshet of the University of British Columbia. As well, the society’s president-elect is Canadian Mark Lewis, who is returning shortly to the University of Alberta from the University of Utah. Also of note, Quebec’s Centre de recherches mathématiques is devoting the 2000-01 academic year to the rapidly developing field of mathematical methods in biology and medicine through workshops covering various applications of nonlinear dynamics in biology, medicine, genomics and medical imaging.

Industry is becoming increasingly interested in such work, which has led to the creation of the Mathematics and Information Technology and Complex Systems Network of Centres of Excellence, or MITACS, headquartered at Simon Fraser University. Dr. Glass’ work with cardiac arrhythmias is just one of many projects funded by the network.

MITACS officially began in February 1999, and “from day one we've put a heavy emphasis on industrial participation in our projects,” says Arvind Gupta, MITACS program leader and a mathematics professor at SFU. The network founders weren’t sure at first how receptive industry would be to working with mathematicians from academia, but the partnership has been a success – MITACS has attracted 78 industrial partners, “by far the largest number of any NCE,” Dr. Gupta says.

## Industry outreach

There was also the opposite concern, that mathematicians might not be too interested in doing “industrial outreach kind of work”, as Dr. Gupta puts it, but this has equally proven to be unfounded. “People were very, very receptive and wanted to find out what this kind of work was all about. We now have some 230 scientists getting funding from MITACS,” Dr. Gupta notes. In fact, one scientist’s whole research program is now based on the work he’s doing with MITACS, he says.

This demand “has allowed us to look carefully at which companies we want to work with. Which companies are offering us intellectually stimulating projects that the students are going to gain by working on. We don’t want to be in the position of doing just consulting work for industry,” he says. A committee of 30 eminent scientists evaluates MITACS project proposals, “and a big part for us of a project succeeding is that it develops new and interesting mathematics”.

As with many Canadian scientists, Dr. Gupta says he is concerned about the number of students who leave the country upon graduation. An organization like MITACS “can show companies the benefit of hiring highly qualified people, and then those people will stay if they see job opportunities here," he says. "We've had companies, for example, turn around [after their participation with MITACS] and create positions where mathematics research is a big aspect of the position. That's heartening to see because those are the kinds of positions that students are attracted to and make them want to stay in the country."

MITACS receives about $3.6 million a year from Canada's three federal granting agencies and Industry Canada, while Canadian industry brought in close to $1.5 million in matching funds last year. With some additional provincial grants, Dr. Gupta expects to have a budget of close to $5.5 million for this year.

### New generation

Keith Promislow, a colleague of Dr. Gupta's at SFU, says there is "a new generation of applied mathematicians coming out . . . [who are] much more eager to reach out and much more eager to redevelop ties". The previous generation, from around the mid-1960s to the early 1980s, "really didn't want to be bothered having to deal with the messiness" of industrial-type problems, but these offer a real learning opportunity, he says. "You're forced to confront what gets in your path, whereas if you're doing isolated research you may meander along the path of least resistance and do what people have done before."

When working with industry, he says, mathematicians often find themselves reaching across to other disciplines, such as engineering or chemistry, "to accumulate a base of knowledge that otherwise would have been available only in fractured pockets. You really have to become multidisciplinary to do industrial math."

Dr. Promislow says he can't give enough credit to MITACS. "It's been a real shot in the arm. It has greatly facilitated the possibility to do this. We get money at our disposal with relatively little administrative overhead, and that's so important. There is money [from other funding sources] that I won't even consider going after because it's just too much work to get, too much work to manage, and too much reporting."

Dr. Promislow is currently working on a MITACS project with Ballard Power Systems, the makers of the Ballard fuel cell, which may one day run most of our automobile engines. "We're trying to model what happens inside of the fuel cells, particularly what's called the gas diffusion layer," he says.

The design parameters of the fuel cell "are so huge, you get lost in all the complexity," he says. "We're sort of like the Marines. If Ballard has a problem or issue, we go in and focus on this thing and try to solve it."

Michael Kouritzin, another MITACS member and a mathematics professor at the University of Alberta, says in the past mathematics "has had a bad rap, partly of its own doing". Mathematics "used to be the science of hard problems", he says. "Then, because of intellectual appeal or whatever, there was this large deviation and real-world applications became irrelevant for a long time. Now there's a shift back to more balance."

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Dr. Kouritzin works with Lockheed Martin Canada to develop mathematical theories for computer tracking systems. At a busy airport, he explains, the number and positions of aircraft, and other variables like local weather and the effect of a plane's wake on other planes, are features of an "interacting stochastic system", and these variables severely complicate the modeling, tracking and prediction procedures.

The mathematical theory being developed in the project "will reach far beyond the air traffic management problem to modeling, prediction and detection problems in narcotics smuggling, search, surveillance and military applications," Dr. Kouritzin wrote in his MITACS proposal. The resulting research has been "kind of fun," he says, "in the sense that theory actually applies. Implementation algorithms have actually proven extremely useful to prove theoretical results."

In another MITACS project, Luis Seco, a mathematics professor at the University of Toronto, is developing mathematical tools for the measurement and management of financial risk. This type of work dates back to the 1970s, when mathematicians realized that certain mathematical equations have relevance to the understanding of the pricing and risk management of complicated financial instruments, typically options and derivatives, which were popular at the time.

More recently, Dr. Seco has been applying this type of analysis to the California electricity market, which has been suffering from power shortages following deregulation due to high demand and a lack of generating capacity. "A lot of those problems have to do with the inability of the market to come up with the right price for, in this case, a new type of security, which is electricity," he says. "Electricity is like a new financial instrument in the sense that its price fluctuates like any other commodity."

This has created a risky market in what was before a very safe market, he says. Because of that, people take actions as a function of their risk aversion, and in California, "this has had very negative effects." Alberta has also deregulated its electricity market, and Ontario is expected to soon follow.

Dr. Seco was also instrumental in the creation of RiskLab at U of T's Mississauga campus. Established in 1996, it helps clients, including financial institutions like Scotiabank and the Bank of Montreal, to develop solutions to a wide variety of risk management problems. While developing solid industry relationships and generating income for the university, RiskLab has also created valuable research and training opportunities for faculty and students – a lot of the work ends up becoming his graduates' PhD theses, says Dr. Seco. Based on the U of T model, seven separate RiskLabs have opened around the world, including those at the University of Cambridge, Cornell University and University of Texas.

Dr. Seco says he is one of an increasing number of people "who are becoming convinced that mathematics is going into a golden age right now". Graduate students who perhaps 10 years ago were not eager to pursue an academic-only career are now finding that their degree is increasingly welcome by industry, adding variety "to what may have been regarded a long time ago as a very one-dimensional profession." Simon Fraser's Dr. Promislow agrees: "It's an exciting time to be a mathematician."